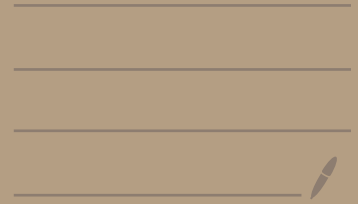
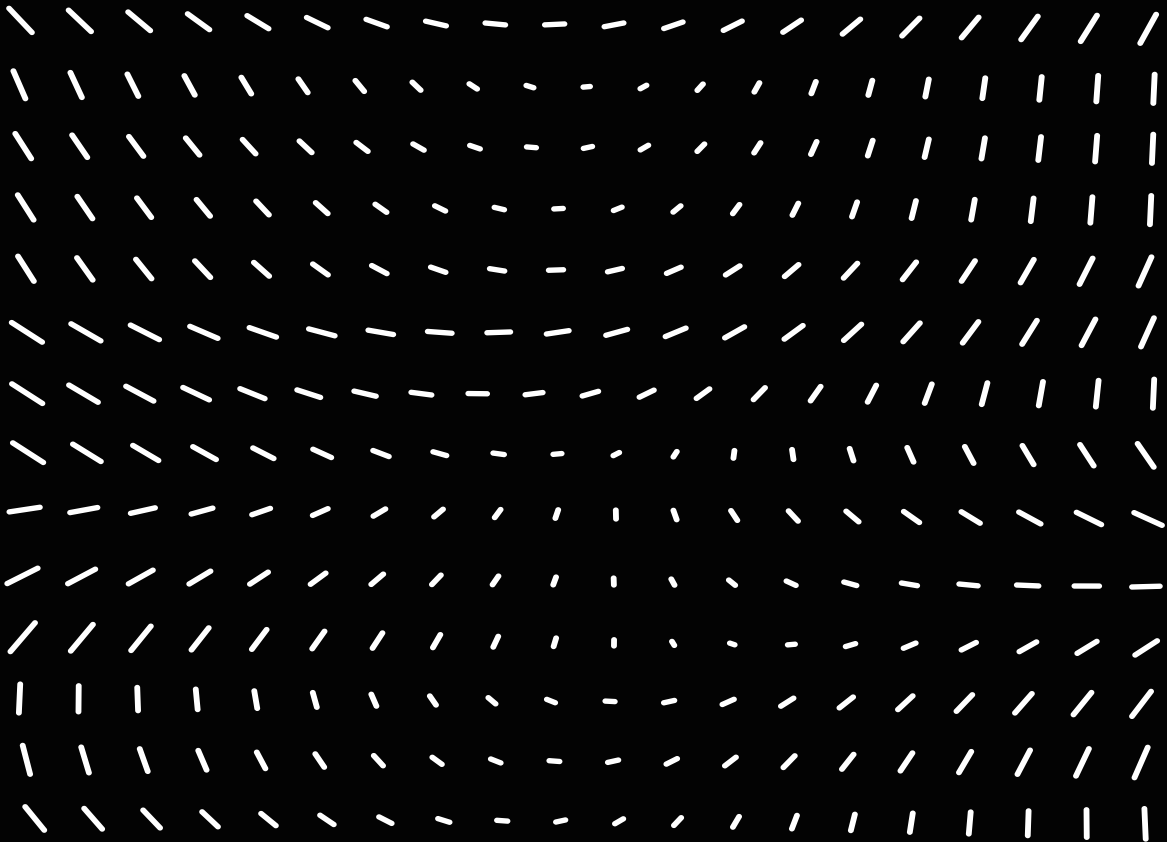


# CISC 221

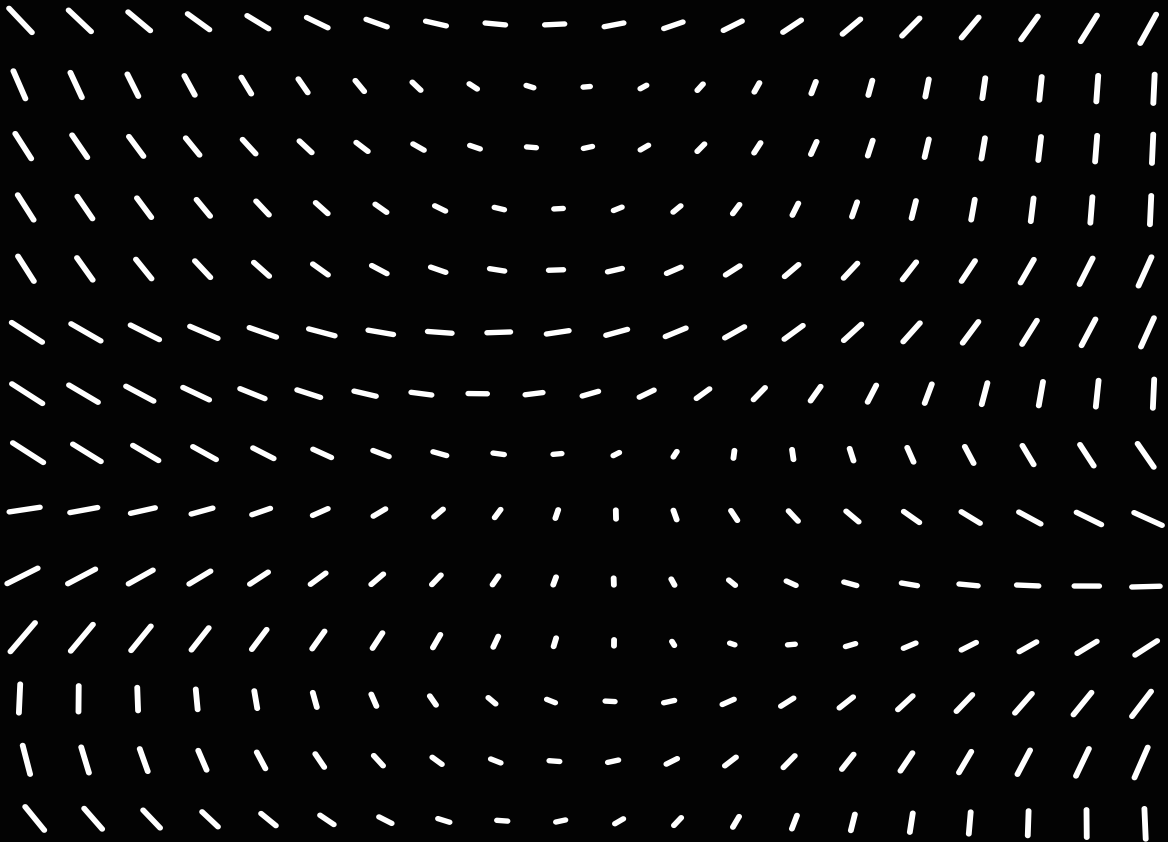
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# Lecture Notes



# Unit 1



# Lecture 1 - Introduction

## Bit about C-language

- C - Structural programming language
- files end in .c
- main function is required
- must be compiled before they run
- lower level language, communicates with assembly and machine code.
- C program, compiler translates to assembly code, which processor understands as machine code.
- C has efficient compiler, preferred choice for system level programming

## Assembly Advantages

- Debugging
- Performance Tuning
- Reverse Engineering

slides 8-10

## Interesting Notes

- Computers can only store numbers in a limited discrete format
- Common Problems with integer overflow and float precision

### Computer arithmetic:

- 1) Finiteness - Numbers stored using fixed bits
- 2) Discreteness - operations are restricted to ranges

### Memory Matters:

- Memory is not infinite or uniform
- 1) Cache and RAM - Data is easier for fast access
- 2) memory bugs - Accessing invalid memory, causes crashes
- C and C++ does not provide memory protection.
- Memory access pattern significantly affect performance

## Holistic Interview:

High Level Programming → Assembly → Machine Code → Execution

### Memory Access:

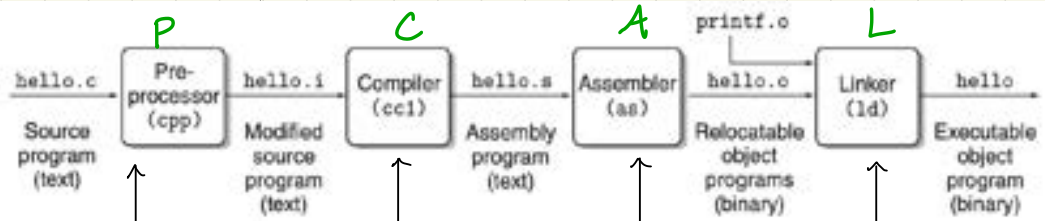
- Hierarchical structure (cache, RAM)
- Speed efficiency

Refresh: Life Cycle of a C Program

- 1) Write source code in C file
- 2) Compiling: Translates source code to machine readable instructions
- 3) Linking: Combine your code with libraries to executable file
- 4) Execution: Run program as binary on operating system

From Source to Binary Code

Compilation Phase:



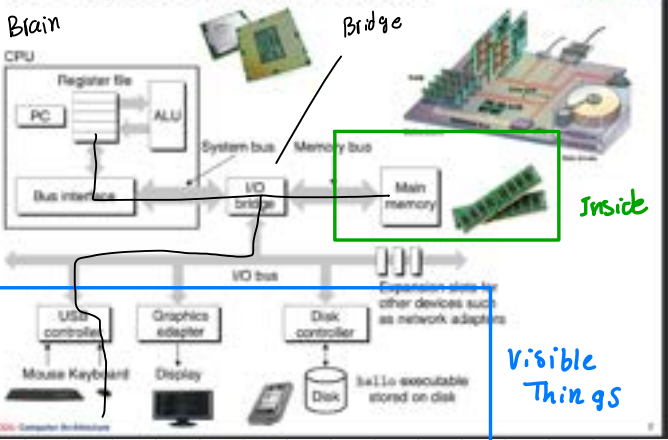
#Includes directives macros and comments

Converts C into Assembly Code

Converts Assembly to machine Code

Combines object files into an executable file

HW Organization of a System



For Hello: Read Command (Shell captures input, writes to memory, triggers program execution)

For Hello: Binary Command (DMA - Direct Memory Access loads Binary File into memory for execution)

# Course Focus

- Hardware Management: OS prevents misuse, simplify app dev, manage concurrency and parallelism.

## C Refresher

- Strongly Typed Language, has strict data type rules
- Basic Data Types: char, int, float, double, short, long  
Big 4
- Strings are arrays of char
- Type Casting: `int height = 2;`  
`float fheight = (float)height`

## Structs:

- A compound datatype for grouping related variables
- Access elements using dot operator

EX

```
struct Point {
    float x;
    float y;
};
typedef struct Point Point;

Point myPoint;
myPoint.x = 4.3;
myPoint.y = 7.1;

// struct myPoint(4.3, 7.1);
```

## Pointers

- Variable storing the memory address of another variable
- Dereference a pointer using `*`: `*ptr = 10;` assigns 10 to x
- Be careful, for memory leak

## Arrays

- Contains multiple elements of same type
- Size is specified at declaration
- Square brackets for indexing

```
arrSize = 3;
int arr[3];
arr[0] = 12;
arr[2] = 34;

int arr2[] = {12, 34, 56};
```

## Operators

All derived from C: `+`, `-`, `/`, `*`, `++`, `--`,  
`<`, `>`, `||`, and symbol

- Make sure to cast correctly with these

## Precedence and Associativity

Precedence: Determines order of evaluation (Bedmas)

Associativity: Determines order when precedence is equal

## Flow Control

- Flow statements: if, if/else, while, do, for, switch/case

- C Does not have boolean types, instead 0 means false, else true

- Equality is ==

## Functions

- use functional decomposition

- take arguments/return values

- Variable declared inside functions have local scope.

```
int square (int i) {  
    return i * i;  
}
```

## Program File Structure

.C - Code files

header files (.h) - has functions, prototypes, macros

- use #include, so files can see each other

- C has a large number of built in library files (.h files) and compiled code

# Lecture 3 - Bits and Bytes

## Hardware Organization Recap

- Memory is organized hierarchically (registers, cache, RAM)
- Instructions and data are fetched from memory to be executed

## Bits

- A bit is the smallest unit of data in a computer (represents 0, 1)
- Bits are building blocks for encoding data (numbers, characters, instructions)
- Bits are easy to implement using bistable elements (0 = low voltage, 1 = high voltage)
- Bits enable universal representation for everything

EX

2 Bits can represent 4 combinations  
00, 01, 10, 11

Text: Each letter A or B is turned into  
a series of bits  
"A" in Binary: 01000001

## Bits and Binary

- A bit is one binary digit (0 or 1)
  - Binary can encode numbers:
- Each bit is a place ( $2^2$ ) + ( $2^1$ ) + ( $2^0$ )  
in binary system      4 + 0 + 1 = 5

## Memory

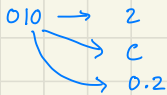
- Memory is structured by addresses and corresponding data
- Address: unique location
- Data: stored information (binary)

## Data Representation and Interpretation

- Binary data represents different types of information based on context.  
(could be Numerical, Alphabetical, other)

EX

Binary could mean:



## Binary and Friends

- Radix (Base) - The base determines the number of unique digits to use.  
Binary (base 2) → 0, 1  
Decimal (base 10) → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9  
Hexadecimal (base 16) → 0-9, A-F

## Binary and Friends - Representation

Each position in a number corresponds to a power of the base

$$\text{Value} = d_0 \times b^0 + d_1 \times b^1 + d_2 \times b^2$$

where  $d_i$  is the digit and  $b$  is the base.

## Conversion of Bases

Ex 1 Convert 1001011011 to decimal (Binary to decimal)

$$1 \times 2^9 + 1 \times 2^6 + 1 \times 2^1 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 \\ = 603$$

Note: 1 hex digit = 4 bits

Ex 2 Convert 50 to Binary (Integer to Binary)

$$50 \rightarrow 2^5 + 2^4 + 2^1 \\ 32 + 16 + 2 \\ = 110010 \text{ in Binary}$$

Ex 3 Hexadecimal to decimal

25B  $\rightarrow$  decimal

$$2 \times 16^2 + 5 \times 16 + 11 \times 16^0 = 603$$

Ex 4 Binary Fraction to decimal

10.0101

$$1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) \\ = 2.3125$$

Ex 5 Hexadecimal to Binary Conversion

2F  $\rightarrow$  Binary

2 in hex = 0010

F in hex = 1111

} 2F = 00101111

Ex 6 Convert Binary to Hexadecimal

11011101

1101 = D

1101 = D  $\rightarrow$  Hex = DD

## Bytes

• A byte is 8 bits

Represents values from:

Binary: 00000000 to 11111111

Decimal: 0 to 255

Hexadecimal: 00 to FF

Hexadecimal Representation in C:

- 0xFA1D37B represents the hex value
- Each digit corresponds to 4 binary bits  
F = 1111, A = 1010

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

## Byte-Oriented Memory organization

- Each memory has an address and content
- Pointers: store memory addresses and have a specific type associated with them.

## Machine Words

- Word size is the nominal size of an integer-valued data (determines the size of virtual memory address space.)  
[Typical values: 32-bit/64 bit]

- EX
- 32 bit system can address  $2^{32}$  bytes = 4 GB
  - 64 bit system supports  $2^{64}$  = 18 exabytes

## Data Representation

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	8	8
float	4	4	4
double	8	8	8
long double	-	-	10/16
pointer	4	8	8

Note: Even if a variable is undefined it takes up the same amount of space in memory

## Strings in C

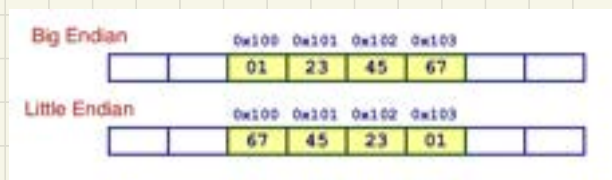
- Strings are arrays of characters
- Each character is encoded in ASCII (7-bit standard)
- Strings are null terminated

## Word-Oriented Memory Organization:

- A word is a fixed size unit of data used by a computer (4 bytes or 32 bits)
- Memory is structured in bytes, but CPU reads and processes data in words.
- Each byte memory has unique address

## Byte Ordering

- Big Endian: Stores most significant byte (MSB) at smallest address.
- Little Endian: Stores least significant byte (LSB) at the smallest address



## Representing Integers

- Integers can be stored in binary, hexadecimal or decimal
- Little Endian: 6D 3B 00 00 } For systems
- Big Endian: 00 00 3B 6D
- Disassembly is translating machine code into human readable assembly code.

Ex

A value 0x12AB in little endian

raw memory layout: AB 12 00 00

This means least significant byte AB comes first

Address	Instruction Code	Assembly	Re rendition
8048365:	5b	pop	%ebx
8048366:	81 c3 ab 12 00 00	add	\$0x12ab, %ebx
804836c:	83 bb 28 00 00 00 00	cmpl	\$0x0, 0x28(%ebx)

0x12ab Value  
0x000012ab Padding to 32 bits  
00 00 12 ab Represent into bytes  
ab 12 00 00 Little endian representation

## How Memory layout is displayed

```

Result (Linux x86-64):
int a = 12345;
Address location: 216e6bf310 Value: 39
Address location: 216e6bf31d Value: 30
Address location: 216e6bf32e Value: 00
Address location: 216e6bf33f Value: 00

```

## Representing Pointers

- A pointer stores the memory address of a variable
- Different compilers assign different locations to objects.

## Representing Strings

- Strings are array of characters, represented by 1 byte (4 bits) (ASCII encoding)
- Byte ordering doesn't affect strings

## Little Endian and Big-Endian Store Numbers

- When a number is stored in memory, its binary representation is broken into bytes (8 bit groups)

Ex

Decimal: 15213 → Binary: 0011 1011 0110 1101

Break into bytes:

- First byte: 0110 1101 → hex: 6D
- : 0011 1011 → hex: 3D
- : 0000 0000 → 00
- 0000 0000 → 00

Big endian: 00 00 3D 6D

Little endian: 6D 3D 00 00

Storage: 0x16e6bf31c  
0x16e6bf31d

memories in address

## Steps in Endian Conversion

- 1) Convert to binary
- 2) Break into bytes
- 3) Rearrange bytes depending on Big/Little Endian

# Lecture 4 - Integers

## Quick Recap

XP - Points to memory location  
\*XP - Points to actual value

- Variable like `int x`, `long y` are stored in memory with specific addresses
- Pointers (`int *xp`, `long *yp`) stores address of variables.
- If `x` starts at an address `0x100`, next `int` would stored next 4 bytes at `0x104`.

## Logical operators

- AND (&): True if both inputs are 1 (0 and 1 = 0)
- OR (|): True if at least one input is 1.
- NOT (~): Inverts the bit 0 to become 1.
- XOR (^): True if inputs are different
- Note: (1) means True, (0) means False
- Can be represented using switches

## Bit Level Operations in C

- C supports bitwise operations to integral datatypes (`long`, `int`, `short`, `char`)
- Operations are applied bit by bit.

EX

- 1) `~0x41`: Inverts each bit `01000001` → `10111110`
- 2) `0x69` and `0x55`

## Logical vs. Bitwise Operations

- Logical operators (`&&`, `||`, `!`)
  - Treat 0 as false, non zero as true.
  - return 0 or 1
- Bitwise operators (`&`, `|`, `~`, `^`)
  - Operate on the binary level

# Unsigned Integers / Encoding Integers

- Unsigned integers only represent non-negative numbers
- stored as binary
- Signed integers use sign bit to rep positive/negative

0: Positive  
1: Negative

$$\text{Unsigned: } B2U(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

- $x_i$ : The bit at position  $i$
- $w$ : Total num of bits

## Two's Complement

$$B2T(x) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

- Key idea is representing neg values by flipping bits adding 1

- Different data types have varying ranges, depending on signed/unsigned

## Binary Number Property

- Binary follows base 2 positional system
- unsigned: 0 to  $2^w - 1$
- Signed:  $-2^{w-1}$  to  $2^{w-1} - 1$

## Two's Complement $\rightarrow$ when u have neg to pos

- How negative numbers are stored:
  - Flip all bits
  - Add 1 to result

Ex 1: 5: 0000101  
-5: 1111010 Flip  
Add 1: 1111011 Add 1

Ex 2: Positive: 0011011 0110101  $\rightarrow$  15213  
Negative: 11000100 10010010  $\rightarrow$  -15213

Note: Larger word sizes allow for larger ranges

More bits means more range

## Modular Arithmetic

- Numbers wrap around after reaching max value ( $2^w$ )
- Unsigned Arithmetic
  - operates within 0 to  $2^w - 1$
  - Adding / Subtracting beyond this range, wraps around

## Signed Arithmetic (Two's Complement)

- operates within  $-2^{w-1}$  to  $2^{w-1} - 1$

# Shifting operations

## Left Shift ( $x \ll y$ )

- Moves bits of  $x$  to the left by  $y$  positions
- Fills right side bits with 0
- Multiplies the number by  $2^y$  (if no overflow)

EX

Binary: 0010 (2)

Shift 2 Left  $\rightarrow$  1000 (8)

## Right Shift ( $x \gg y$ ):

- Move bits of  $x$  to right by  $y$  positions
- **Logical Shift:**
  - Fills left bits with 0    1010 (2)  $\rightarrow$  0010
- **Arithmetic Shift:**
  - Fills left bits with **Sign bit**    1110  $\gg 2 \rightarrow$  1111

Special case: When the amount  $k > w$  (where  $w$  is word size) behaviour is undefined in C but usually wraps around  $k \bmod w$ .

## Left Shifts

$x \ll 3$

- Moves all bits in binary to left by 3 position
- Fills right with 0's

0xC3  $\rightarrow$  1100 0011  $\rightarrow$  00011000  $\rightarrow$  0x18  
For  $x \ll 3$

## Right Shift

- moves bits to right by  $n$  positions
- Logical fills left most bits with 0's
- Arithmetic fills with MSB

# Lecture 5 - Integer Arithmetic

B2U(x) - Unsigned  
B2T(x) - Signed  
MSB - Most Significant bits

## RECAP - Numeric Datatypes

- Signed Integers: represent positive/negative values

EX int (4 bytes), short (2 bytes)

- Unsigned Integers: represent positive values, and 0

EX unsigned int (4 bytes)

- Real Numbers:

EX Floating-point values like float (4 bytes) and double (8 bytes)

### Example 1

- 8 bit binary: 11111010

• Unsigned: B2U = 250

• Signed: B2T = -6

## Conversion and Casting

- Key Ideas: Bit patterns are the same for both signed and unsigned numbers, but interpretation varies.

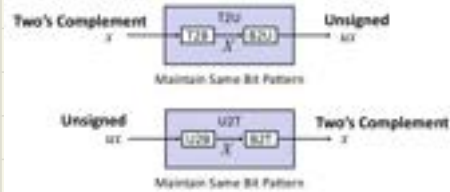
## Mapping Between Signed/Unsigned

- signed to unsigned (T2U)

• retain bit pattern, interpret as unsigned

- unsigned to signed (U2T)

• retain bit pattern, interpret as signed



• Mappings between unsigned and two's complement numbers:  
Keep bit representations and reinterpret

## Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11

Arrows indicate the mapping: T2U (Signed to Unsigned) and U2T (Unsigned to Signed).

# Mapping Signed/Unsigned

If you had 4 bits, instead of 8,  
 unsigned values, would go up to 15, (1111)  
 signed values, would go from -8 to 7.

Ex  
 To find range, use 1111 = -8 (two's complement)  
 0111 = 7 (Positive)

Neg range: 1000 to 1111 (-8 to -1)

Pos range: 0000 to 0111 (0 to 7)

- A large unsigned number maps to a negative signed number

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13

Note: largest negative weight in signed, becomes largest positive weight in unsigned.

## Signed vs. Unsigned in C

- Constants by default are signed integers unless specified with U at end
- Casting is explicit, manually done to convert between signed/unsigned

int tx = (int) ux;    cast Unsigned → Signed  
 unsigned ty = (unsigned) ty;    cast Signed → unsigned

## Sign Extension

- extending a smaller integer to a larger type, while preserving its value
- Rule: for signed integers (pos or neg):

① Extend the MSB to fill extra bits

Ex Positive number extension:

short value: 15213 → 0011011 01101101

int value: Extended → 0000000 0000000 0011011 01101101

Same applies to negative value

## Truncation

- Reducing a larger integer to a smaller type by dropping the leading bits
- Behaviour is equivalent to taking  $\text{mod } 2^k$  of number for unsigned

## Summary Expanding/Truncating

- Expanding:
  - unsigned: Fills with zeroes
  - signed: Extends with the sign bit
- Truncating:
  - unsigned: Drops leading bits,  $\text{mod } 2^k$
  - signed: Similar to unsigned but reinterprets

Note: If truncation removes bits critical to determining the value, there will be data loss.

## Unsigned Addition

- Operates in modular arithmetic:
  - Sum is calculated as  $x + y \text{ mod } 2^w$ , where  $w$  is bit width
  - True Sum: Calculated as  $x + y$  with  $w + 1$  bits.
  - Modular Sum: Keeps only the lower  $w$  bits, discarding carry bit

### Example

$$x = 1111, y = 0001$$

$$\text{True Sum: } 10000$$

$$\text{Modular Sum: } 0000 \text{ (} 16 \text{ mod } 2^4 \text{) ?}$$

- Wrap around behaviour; if  $x + y \geq 2^w$ , sum wraps around  $x + y - 2^w$ .

True Sum; Values exceed  $2^w$ , but aren't stored fully

Modular Sum; Ranges from 0 to  $2^w - 1$

Overflow when True sum exceeds  $w$  bit range

# Mathematical Properties

## Modular addition forms an Abelian Group

- Closed under addition  
 $0 \leq x +_w y \leq 2^w - 1$
- Commutative  
 $x +_w y = y +_w x$
- Associative  
 $(x +_w y) +_w z = x +_w (y +_w z)$
- 0 is additive identity  
 $x +_w 0 = x$
- Every element  $x$  has an additive inverse  
 $2^w - x$

## Two's Complement Addition

- Signed addition works the same as unsigned at the bit level
  - Operands are  $w$ -bits
  - True sum is  $w+1$  bits
- result is truncated to  $w$  bits by discard carry

## Overflow in Two's Complement Addition

- Overflow happens when the true sum exceeds the signed range  $(-2^{w-1} \text{ to } 2^{w-1}-1)$
- Positive Overflow:  $x+y > 2^{w-1}-1$ , wraps around to negative
- Negative Overflow:  $x+y < -2^{w-1}$ , wraps to positive
- $-x = \sim x + 1$   
Flip all bits  $\nearrow$  add on

## Multiplication Basics

- Multiplying two  $w$ -bit numbers can produce a result that requires up to  $2w$  bits
- Unsigned Multiplication:  
 $0 \leq x \cdot y (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
- Two's Complement Multiplication:  
 $x \cdot y$  ranges from  $-(2^{2w-2})$  to  $(2^{2w-2} - 1)$

## Unsigned Multiplication in C

- True product is calculated as:  
 $x \cdot y \bmod 2^w$
- The high order bits are discarded.

## Signed Multiplication in C

- discards high-order bits but treat numbers as twos complement
- Lower bits are the same for signed and unsigned multiplication at bit level.

## Power of 2 Multiplications

- Multiplication by power of 2 can be done using left shifts:  $x \cdot 2^k = x \ll k$

Ex  $x \ll 3 = x \cdot 8$

$$(x \ll 5) - (x \ll 3) = x \cdot 24$$

- Shifting is faster than multiplication

## Power of 2 Division

- Division by powers of 2 is performed using logical right shifts:  $x \div 2^k = x \gg k$

Ex  $x = 15213$

$$x \gg 1 = 7606$$

$$x \gg 4 = 950$$

## Summary: Basic Rules

### ■ Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod  $2^n$ 
  - Mathematical addition + possible subtraction of  $2^n$
- Signed: modified addition mod  $2^n$  (result in proper range)
  - Mathematical addition + possible addition or subtraction of  $2^n$

### ■ Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod  $2^n$
- Signed: modified multiplication mod  $2^n$  (result in proper range)

# Types of overflow

• Positive overflow, if  $x > 0, y > 0$  and result  $x+y$  is negative.

Ex

$$x = 01100$$

$$y = 00100$$

$$x+y = 10000$$

$$= -16$$

• Negative overflow, happens when two negative numbers are added, result becomes smaller than  $-2^{w-1}$

$$x = 10100, y = 10001$$

$$(-12) \quad (-15)$$

$$-12 + -15 = -27$$

Binary:  $1000101 \rightarrow 00101$ , negative overflow.

# Lecture 6/7 Floating Points

• Floating point numbers represented in the form:

$$V = (-1)^S \times M \times 2^E$$

- S: Sign bit (0 = pos, 1 = neg)
- M: Mantissa [frac part 1.0 - 2.0 usually]
- Exponents

## Normalized Numbers

- E = exp-biased (the exponent is not all 0 or 1's)
- 1. Frac

# Quiz 1 Study guide

High Level Programming → Assembly → Machine Code → Execution

## Pointers

- Variable storing the memory address of another variable
- Dereference a pointer using  $*$ :  $*ptr = 10$ ; assigns 10 to  $x$
- Be careful, for memory leak
- $*p = 20$ ; Assigns value to memory location

## C Refresh

- Strongly Typed Language, has strict data type rules
- Basic Data Types: char, int, float, double, short, long  
Big 4
- Strings are arrays of char
- Type Casting:  $int\ height = 2$ ;  
 $float\ fheight = (float)height$

## Bits

- A bit is the smallest unit of data in a computer (represents 0, 1)
- Bits are building blocks for encoding data (numbers, characters, instructions)
- Bits are easy to implement using bistable elements (0 = low voltage, 1 = high voltage)
- Bits enable universal representation for everything

### Bits and Binary

- A bit is one binary digit (0 or 1)
- Binary can encode numbers:

Each bit is a place  $(2^2) + (2^1) + 2^0$   
in binary system  $4 + 0 + 1 = 5$

## Binary and Friends

- Radix (Base) - The base determines the number of unique digits to use.

Binary (base 2) → 0, 1

Decimal (base 10) → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Hexadecimal (base 16) → 0-9, A-F

A=10, B=11... F=16

Always needs to be 16 bits / 2 bytes

## Bytes

• A byte is 8 bits

Short - 32 bits

long - 64 bits

Represents values from:

Binary: 00000000 to 11111111

Decimal: 0 to 255

Hexadecimal: 00 to FF

Hexadecimal Representation in C:

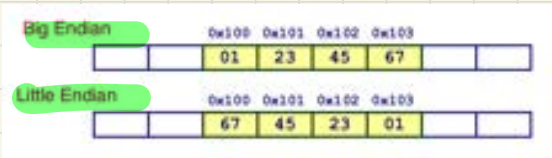
- 0xFA1D37B represents the hex value
- Each digit corresponds to 4 binary bits  
F = 1111, A = 1010

## Strings in C

- Strings are arrays of characters
- Each character is encoded in ASCII (7-bit standard)
- Strings are null terminated

## Byte Ordering

- Big Endian: Stores most significant byte (MSB) at smallest address.
- Little Endian: Stores least significant byte (LSB) at the smallest address
- Endianness: Determines how multi-byte data is stored in memory



## Little Endian and Big-Endian Store Numbers

- When a number is stored in memory, its binary representation is broken into bytes (8 bit groups)

Ex: Decimal: 15213 → Binary: 0011 1011 0110 1101

Break into bytes:

- First byte: 0110 1101 → hex: 6D
- : 0011 1011 → hex: 3D
- : 0000 0000 → 00
- 0000 0000 → 00

Big endian: 00 00 3D 6D

Little endian: 6D 3D 00 00

### Steps in Endian Conv

- 1) Convert to binary
- 2) Break into bytes
- 3) Rearrange bytes depending on Big/Little Endian

## Bitwise operators

- AND (&): True if both inputs are 1 (0 and 1 = 0)
- OR (|): True if at least one input is 1.
- NOT (~): Inverts the bit 0 to become 1.
- XOR (^): True if inputs are different
- Note: (1) Means True, (0) Means False
- Can be represented using switches

## Bit Level Operations in C

- C supports bitwise operations to integral datatypes (long, int, short, char)
- Operations are applied bit by bit.

EX

- 1) ~0x41: Inverts each bit 01000001 → 10111110
- 2) 0x69 and 0x55

## Logical Operations

- Logical operators (!, ||, !) → use double signs
- Treat 0 as false, non zero as true.
- return 0 or 1
- Uses ! instead of ~

## Unsigned Integers / Encoding Integers

- Unsigned integers only represent non-negative numbers
- stored as binary
- Signed integers use sign bit to rep positive/negative

0: Positive  
1: Negative

$$\text{Unsigned: } B2U(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

- $x_i$ : The bit at position  $i$
- $w$ : Total num of bits

## Two's Complement

$$B2T(x) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

- Key idea is representing neg values by flipping bits adding 1

## Binary Number Property

- Binary follows base 2 positional system
- unsigned: 0 to  $2^w - 1$
- Signed:  $-2^{w-1}$  to  $2^{w-1} - 1$

## Two's Complement $\rightarrow$ when u have neg to pos

- How negative numbers are stored:
  - Flip all bits
  - Add 1 to result

Ex 1: 5: 0000101

-5: 1111010 Flip

Add 1: 1111011 Add 1

Ex 2 Positive: 0011011 0110101  $\rightarrow$  15213

Negative: 11000100 10010010  $\rightarrow$  -15213

Note: Larger word sizes allow for larger ranges

More bits means more range

## Modular Arithmetic

- Numbers wrap around after reaching max value ( $2^w$ )

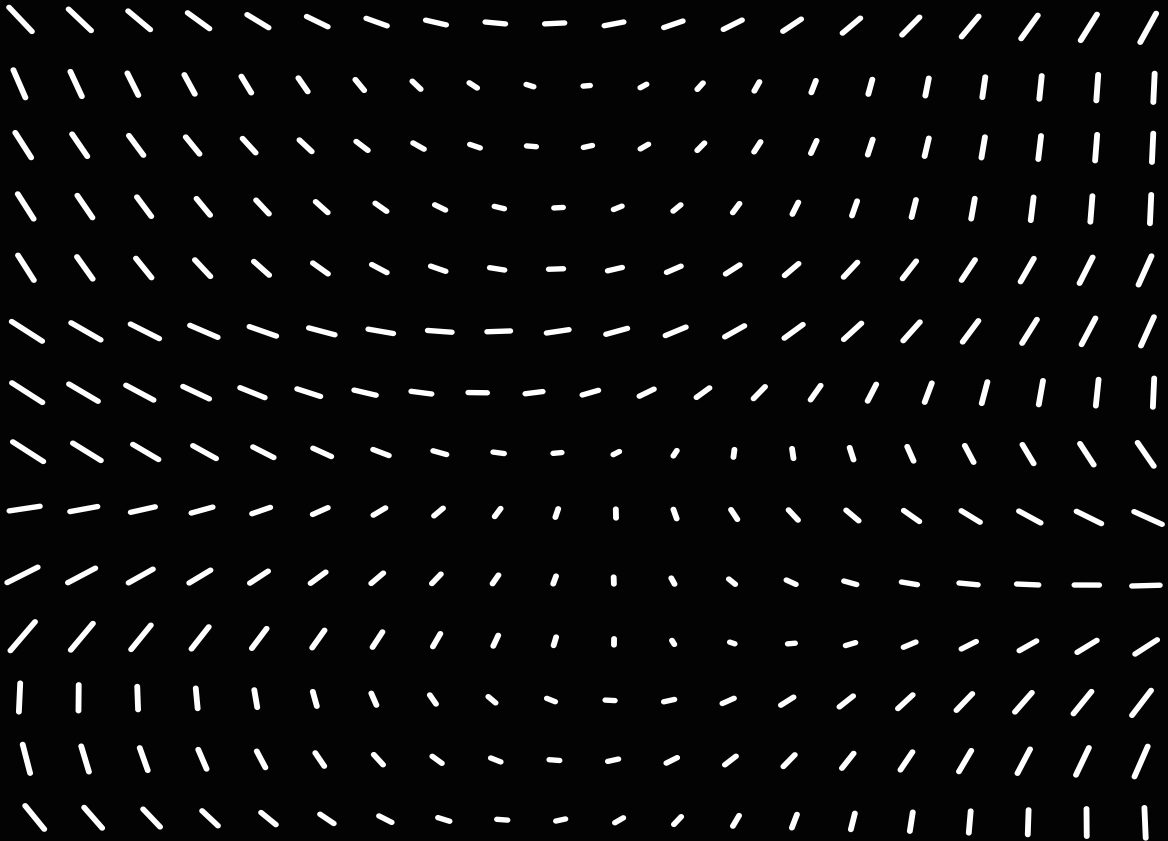
### Unsigned Arithmetic

- operates within 0 to  $2^w - 1$
- Adding / Subtracting beyond this range, wraps around

### Signed Arithmetic (Two's Complement)

- operates within  $-2^{w-1}$  to  $2^{w-1} - 1$

# Unit 2



# Lecture 9 - Machine Fundamentals

## Importance of Machine Code

- 1) Optimize efficiency, understanding how Compilers generate code
- 2) Analyze run time behaviour, see low level execution
- 3) Improve security, identify vulnerabilities

## C, Assembly, Machine Code

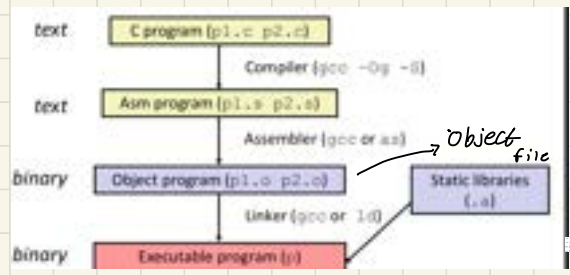
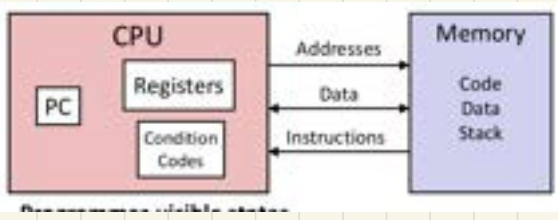
High Level C → Assembly → Machine Code

- Starts with high level C-code
- Then compiler translates C to assembly language (human readable version of machine code)
- Machine Code (Binary) → Assembly is converted into Binary machine code executed by CPU.

- Architecture - known as ISA, defines parts of processor needed to write assembly/machine code
- Micro-architecture - physical implementation of ISA

## Basic CPU Components

- PC (Program Counter) - holds address of next instruction to be executed, automatically increments
- Registers - Small fast storage locations, used to hold temporary data. (many diff types)
- Condition Codes (Flags) - Special registers, store info on recent arithmetic/logic



## Compilation Process

- To convert .C file to an executable
- 1) compilation: Translates C into assembly using gcc
- 2) Assembly: Converts to machine code
- 3) Linking: Combines object code with libraries to make executable program

# Compiling to Assembly

- assembly is annotated by passing parameters, functions
- details size and data type, specifies size in memory

## Assembly Operations

- **Data transfer**: moving data between memory / registers
- **Arithmetic Functions**: Perform calculations using data / memory
- **Control Transfer**: managing flow of execution with in code (branches / conditionals)
- **Assembler**: Translates assembly instructions into binary object code
- **Linker**: Finalizes creation of an executable

C declaration	Intel data type	Assembly code suffix	x86-64 size (bytes)
char	Byte	b	1
short	Word	w	2
int	Double word	l	4
long int	Quad word	q	8
long long int	Quad word	q	8
char *	Quad word	q	8
float	Single precision	s	4
double	Double precision	d	8
sses (untyped pointers)			

## Assembler

- Translates .s into .o, which is binary representation of assembly code.
- Includes binary encoding
- Linking, process of creating executable

## Assembler / Machine code Role

- Assembler Role: Translates assembly code into binary object code, this machine code is what the computer's CPU executes.
- Machine code: Holds the binary data that instructs the CPU on what to do (highly optimized for architecture it's meant to run on)

## Machine Instruction Ex

\* `dest = t;` stores value t, designated by dest

`movq %rax, (%rbx)`

Highlights storing value from one register (%rax) into memory location pointed by %rbx.

`0x40059e; 48 99 03`

- Object code → disassembled back into assembly code

## Registers, Operands and Move

- White side shows full 64 bit-size  
%rax is the accumulator register
- Gray part: Shows register 32-bit, 16-bit, 8 bit.
- Backwards compatibility, each 32-bit register

### x86-64 Integer Registers

%rax	%eax	%r8	%r8d
%rbx	%ebx	%r9	%r9d
%rcx	%ecx	%r10	%r10d
%rdx	%edx	%r11	%r11d
%rsi	%esi	%r12	%r12d
%rdi	%edi	%r13	%r13d
%rsp	%esp	%r14	%r14d
%rbp	%ebp	%r15	%r15d

\* Can reference low-order 4 bytes (also low-order 2 & 1 bytes)

## Moving Data

- movq: instruction used to move data from one place to another
- Could move registers, constants or memory locations

## Operand Types

- Immediate: Direct numerical values prefixed with a \$. Constants like 0x400
- Register: Using registers like %rax or %r13 to store/retrieve values
- Memory: Refers to data in memory specified by register.
- Source/Destination. Source is where it comes from, Destination is where the data is moved to.

## Operand Combinations

- Immediate → register, Same as assigning constant value to variable.
- Register → Register, Copying one variable to another
- Memory → memory, similar to dereferencing pointer in C
- Register → Register, similar assigning value to a dereferenced pointer in C

Note: Memory → memory transfer is not possible, first move data from memory to register then register to another memory.

## Memory Addressing Modes

- Normal Addressing, register is used to hold the address, of the data in memory. (not data itself)
- In C, its dereferencing a pointer to access value it points to
- Displacement, involves register, constant. Where register contains base memory address and constant is an offset added to that address.
- In C, its like accessing array element, base = start, displacement = index

## Understanding swap()

- Registers contain addresses, which point to a value in memory
  - `%rax, (%rsi)`
    - `swap()` uses registers to temporarily hold values in memory addresses given by `%rdi` and `%rsi`
- ↑ takes value at this register
- ↑ points to memory address of this and puts value there

# Lecture 10 - Machine Fundamentals

- addresses should always be 64 bits
- rax is for returning values
- General Format:  $D(Rb, Ri, S)$ 
  - D - displacement, constant value added to address
  - Rb - Base register, holds the base memory address
  - Ri - Index Register, used to calculate offset from base
  - S - scale, multiplier used with Ri

## Address Computations

Expression	Address Computation	Address
<code>0x8(%rdx)</code>	$0xf000 + 0x8$	<code>0xf008</code>
<code>(%rdx,%rcx)</code>	$0xf000 + 0x100$	<code>0xf100</code>
<code>(%rdx,%rcx,4)</code>	$0xf000 + 4 * 0x100$	<code>0xf400</code>
<code>0x80(%rdx,2)</code>	$2 * 0xf000 + 0x80$	<code>0x1e080</code>

## Arithmetic and Logic Operations

- `leaq Src, Dest`; Calculates an address using given addressing mode, stores it **without accessing memory**
- Useful for complex addresses, `leaq(%rdi,%rsi,2),%rax` →  $\text{address} = \text{val in rdi} + 2 * \text{val in rsi}$

### Two-operand instructions:

Format	Computation
<code>add Src, Dest</code>	$\text{Dest} = \text{Dest} + \text{Src}$
<code>subq Src, Dest</code>	$\text{Dest} = \text{Dest} - \text{Src}$
<code>imulq Src, Dest</code>	$\text{Dest} = \text{Dest} * \text{Src}$
<code>salq Src, Dest</code>	$\text{Dest} = \text{Dest} \ll \text{Src}$
<code>sarq Src, Dest</code>	$\text{Dest} = \text{Dest} \gg \text{Src}$
<code>shrq Src, Dest</code>	$\text{Dest} = \text{Dest} \gg \text{Src}$
<code>xorq Src, Dest</code>	$\text{Dest} = \text{Dest} \wedge \text{Src}$
<code>andq Src, Dest</code>	$\text{Dest} = \text{Dest} \& \text{Src}$
<code>orq Src, Dest</code>	$\text{Dest} = \text{Dest}   \text{Src}$

Also called **shlq**  
**Arithmetic**  
**Logical**

### Watch out for argument order!

- shift operations, multiplies register values by multiples of two.
- `imulq` - multiplies two integer values
- `salq` - Shift left, multiplies a value of  $2^{\text{raised to power of shift amt.}}$

## Register Usage

- `%rdi, %rsi, %rdx` used for function arguments
- `%rax, %rdx, %rcx` used for storing intermediate results

Note: Compiler transforms statements/expressions  
Compiler uses many instruction combinations.

# Lecture 11 - Machines Control

## Condition Codes

- Condition codes are special **Single bit flags** that **store the result** of **arithmetic operations**. These codes **control** **conditional flow**, like jumps and stuff.

## Condition Code Registers

- CF (Carry Flag) - set if there is carry from msb. Used for arithmetic
- ZF (Zero Flag) - set if result is 0. Used for equality comparisons ( $x=y$ )
- SF (Sign Flag) - set if result is negative. Used when signed arithmetic
- OF (Overflow Flag) - set if two's complement arithmetic, signed arithmetic overflow detection

Note: Condition codes are automatically updated after an operation like add or sub.

## Explicitly Setting Condition Codes

- `cmpa` - compares two values by seeing if  $(a-b)=0$  `cmpa %rsi, %rdi` ( $rdi-rsi$ )
- `testa` - computes bitwise AND (A&B) `testa %rax, %rax`

## Reading Condition Codes

- 1) setting single byte in register
- 2) conditional branching (jx instructions)
- 3) conditional moves (setx instructions)

## SetX Instructions

- SetX instructions store condition results into a register
- only affects lowest byte of a register

EX

`cmpa %rsi, %rdi`

`setg %al`

`movzbl %al, %eax`

if  $x > y$  then  $\%al = 1$ , else  $= 0$

SetX	Condition	Description
<code>sete</code>	ZF	Equal / Zero
<code>setne</code>	$\sim$ ZF	Not Equal / Not Zero
<code>sets</code>	SF	Negative
<code>setns</code>	$\sim$ SF	Nonnegative
<code>setg</code>	$\sim$ (SF*OF) & $\sim$ ZF	Greater (Signed)
<code>setge</code>	$\sim$ (SF*OF)	Greater or Equal (Signed)
<code>setl</code>	(SF*OF)	Less (Signed)
<code>setle</code>	(SF*OF)   ZF	Less or Equal (Signed)
<code>seta</code>	$\sim$ CF & $\sim$ ZF	Above (unsigned)
<code>setb</code>	CF	Below (unsigned)

# Conditional Branches (JX instructions)

• Conditional jumps that execute based on condition codes

JX	Condition	Description
jmp	1	Unconditional
je	ZF	Equal / Zero
jne	-ZF	Not Equal / Not Zero
ja	SF	Negative
jna	-SF	Nonnegative
jg	-(SF^OF) & -ZF	Greater (Signed)
jge	-(SF^OF)	Greater or Equal (Signed)
jl	(SF^OF)	Less (Signed)
jle	(SF^OF)   ZF	Less or Equal (Signed)
jae	-CF & -SF	Above (unsigned)

EX  
 cmpq %rsi, %rdi # compare x,y  
 jg greater # jump if x > y

• JLE jumps if  $x \leq y$

## Loops

• Loops in Assembly language control repeated execution of code using conditional branches (jx instructions)

### 1) Do while loop

• do while loop executes at least once before checking condition

**C Code**

```
long count_do
(unsigned long x) {
    long result = 0;
    do {
        result += x & 0x1;
        x >>= 1;
    } while (x);
    return result;
}
```

**Goto Version**

```
long count_goto
(unsigned long x) {
    long result = 0;
loop:
    result += x & 0x1;
    x >>= 1;
    if(x) goto loop;
    return result;
}
```

```
movl $0, %rax # result = 0
                # loop:
movq %rdi, %rdx → copy x to rdx, t = x & 0x1, t = x & 1
andl $1, %rdx
addq %rdx, %rax # result += t
shrq %rdi # x >>= 1
jne .L2 # if (x) goto loop
rep; ret
```

Loop → .L2

• Key point: always runs atleast once before if(x) go to loop.

### 2) While Loops

• while loops check the condition before running the loop

**While version**

```
while (!test)
    body
```

**Goto Version**

```
goto: test;
body;
goto goto;
```

**C Code**

```
long count_while
(unsigned long x) {
    long result = 0;
    while (x) {
        result += x & 0x1;
        x >>= 1;
    }
    return result;
}
```

**Jump to Middle**

```
long count_goto_jm
(unsigned long x) {
    long result = 0;
    goto test;
loop:
    result += x & 0x1;
    x >>= 1;
    test;
    if(x) goto loop;
    return result;
}
```

• Uses jump to middle translation

### 3) For Loops

• When you know the amt of times, you want to loop.  
 for(initialize, test, update)

#### "For" Loop → While Loop



# Switch Statements

- allow branching execution based on a variable's value
- In assembly they are implemented using jump table
- large switch statements use a jump table

- Multiple Case Labels → diff case, same execution
- Fall through behaviour → next case
- Missing cases → unreachable?

## Approach

- 1) looks up jump table using  $x$  as index
- 2) Jumps directly to case block
- 3) Executes code for that case

- Use `jmp *` to execute switch case
- Jump table stores addresses of case labels

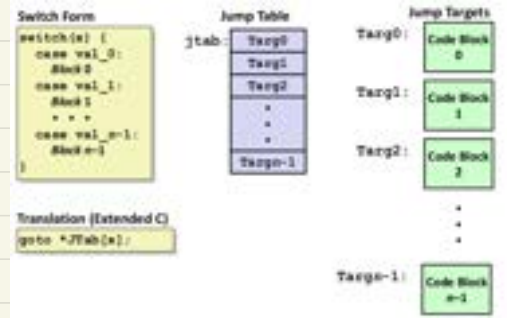
- Base address
- Direct jmp address
- Indirect jmp \*

- Case 1:  $w = y * z$

.L3:  
movq %rsi, %rax #y  
imulq %rdx, %rax #y\*z  
ret

- Cases can have identical behaviour, sharing assembly code

- Each case corresponds to a .L2



# Lecture 12 Machine Procedures

## Mechanisms in Procedures

- Passing control - moving from procedure to procedure
- Passing data - arguments/return values
- memory management - allocate/deallocate memory during execution
- Machine instruction implementation - only uses necessary mechanisms from x86-64

## Stack Structure

- Stack grows towards lower memory addresses
- RSP register points to top of stack (lowest numerical address)
- when data is pushed onto stack, rsp is decremented to → `pushq Src` make room for data at top of stack.
- when data is popped it reads data where rsp points to → `popq Dest` then rsp is incremented

## Passing Control

### C-Code

- multi-store function computes multiplication of x/y using `mult2` then stores results in dest printer

### Assembly Code:

Push/POP Commands save and restore rbp value  
`callq 400550 <mult2>` calls `mult2` function  
Places return address then jumps

- Caller decreases stack pointer to make space
- Caller - prepares function call by pushing to stack
- Callee - executes stack correctly to ensure return address is preserved

## Procedure Data Flow

- registers have first 6 args passed
- return values stored in `%rax`
- stack used for beyond first 6

- 1) arguments set up in registers
- 2) `mult` is called with 2 registers returns in `rax`
- 3) moves result from `rax` to memory located by original pointer

### Code Examples

```
void multstore
(long x, long y, long *dest)
{
    long t = mult2(x, y);
    *dest = t;
}
```

```
0000000000400540 <multstore>:
400540: push    %rax                # Save %rax
400541: mov     %rdi,%rbx           # Save dest
400544: callq  400550 <mult2>      # mult2(x,y)
400548: mov     %rax,(%rbx)        # Save at dest
40054c: pop     %rax                # Restore %rax
40054d: retq                               # Return
```

```
long mult2
(long x, long y)
{
    long a = a * b;
    return a;
}

0000000000400550 <mult2>:
400550: mov     %rdi,%rax          # a
400553: imul  %rdi,%rax           # a * b
400557: retq                               # Return
```

## Stack Discipline

- manage data for limited time
- Each procedure makes new stack frame to allow space for its arguments, local variables, pointers

## Call Chain Examples

- `foo()` → `who()` → `amI()`
- important for recursive functions

## Stack Frames

- Contents of Stack Frames:
  - Return information, return address
  - local storage, Procedures using local vars are stored here
  - Temporary space, used for temp calculations
- Allocation: When a procedure is called new stack frame is made includes space for all of the above (Push by call)
- Deallocation: Stack frame is removed
- Note: In nested procedure calls, a new stack frame is created for each function. `%rsp` and `%rip` gets reprinted.
- When procedures return, the stack is deallocated
  - `ret` uses return function jump back to caller

## Typical x86-64 Stack Frame

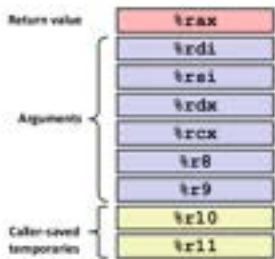
- Current Stack Frame: Contains local variable, parameters for function
- Caller Stack Frame: Consists of return address and arguments for the function

# Register Saving Conventions

- Caller is the function making the call, Callee is one being called
- registers can be for temp storage
- Caller saved registers - caller expects may be modified by the callee and saves them if needed before call
- Callee saved registers - Callee is responsible for saving/restoring

## x86-64 Linux Register Usage #1

- **%rax**
  - Return value
  - Also caller-saved
  - Can be modified by procedure
- **%rdi, ..., %r9**
  - Arguments
  - Also caller-saved
  - Can be modified by callee procedure
- **%r10, %r11**
  - Caller-saved
  - Can be modified by callee procedure



%RSP is special as it points to top of stack.

Caller saved (volatile)  
Callers responsibility to save

non volatile  
must be saved  
restored

## Illustration of Recursion

- $\text{return } (x \neq 1) + \text{fcount} - \text{f}(x-1)$   
adds the 1st bit of  $x$  to result of a recursive call with  $x$  shifted right by 1.
- Recursion unwinds, modifies %rax for return vals
- Functions clean up stack, returns total count of 1 bits

\* Results return in %rax

## Summary

- Each call has private storage on stack including saved register/local variables
- Register saving conventions, crucial to ensure no overwriting.
- Call and return discipline, LIFO, if P calls Q, then Q must complete and return before P
- Mutual recursion.

# Lecture 13 - Machine Arrays

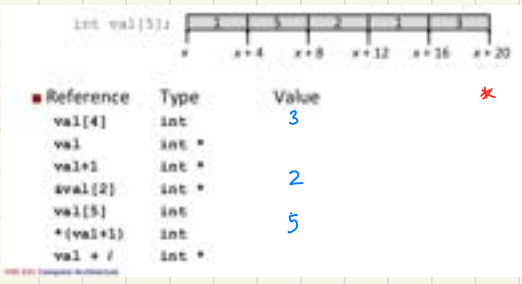
## One-Dimensional Arrays

- Sequence of elements of same type, stored in contiguous memory
- 1-D array  $T A[L]$ , consists of a series of data type  $T$ , length  $L$
- Memory allocation would be  $L * \text{sizeof}(T)$  bytes.

### Ex

char String[12] - 12 bytes  
 int val[5] - spans 20 bytes  
 double a[8] - uses 24 bytes

- Identifier of an array could be used as pointer to first element of array. Base address:  $A[0]$



- typedef int zip\_dig[5] - defines new type repr array of 5 ints
- \* • zip\_dig arr = {1, 5, 3, 1, 2} initializes array of type "zip\_dig".

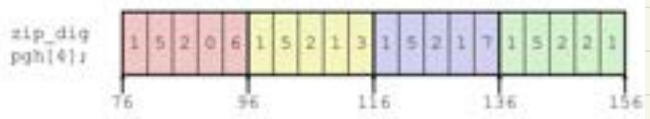
- Register %rdi contains starting address of array
- Register %rsi contains array index

- Array loops have incremental each element. Uses jmps to add to vars and compare each iteration.

## Multi-Dimensional Array

- Declaration:  $T A[R][C]$  :
- R Rows, C Columns
- Type  $T$  elements require  $K$  bytes
- Array Size:  $R * C * K$  bytes
- all elements in row, stored contiguously

```
#define PCOUNT 4
zip_dig pgh[PCOUNT] =
    {{1, 5, 2, 0, 6},
     {1, 5, 2, 1, 3},
     {1, 5, 2, 1, 7},
     {1, 5, 2, 2, 1}}
```



- "zip\_dig pgh[4]" equivalent to "int pgh[4][5]"
- Variable pgh: array of 4 elements, allocated contiguously
- Each element is an array of 5 int's, allocated contiguously

# Accessing Elements

- To access rows  $A[i]$
- Starting address  $A + i * (C * K)$   
Based on row index and size of each row
- Address Calculation:  $A + (i * C + j) * K$  for  $A[i][j]$

- For  $n \times n$  matrixes, can handle easier element access, dynamic resizing
- Fixed 16 by 16 matrix  $a[i][j]$  has memory address  $A + i * (C * K) + j * K$ , where  $C = 16, K = 4$

# Multi-Level Arrays

- Pointers that point to other arrays
  - dynamic/complex ds
- ```
int *univ[Vcount] = {mit, cmu, ucb};
```
- each element in univ points to another array
- Memory Access: involves many dereferencing steps
    - 1) access pointer from base array
    - 2) use pointer to access actual integer

| Nested array                                                                                                | Multi-level array                                                                                             |
|-------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| <pre>int get_pgh_digit<br/>(size_t index, size_t digit)<br/>{<br/>    return pgh[index][digit];<br/>}</pre> | <pre>int get_univ_digit<br/>(size_t index, size_t digit)<br/>{<br/>    return univ[index][digit];<br/>}</pre> |
|                                                                                                             |                                                                                                               |
| Accesses looks similar in C, but address computations very different:                                       |                                                                                                               |
| $Mem[pgh + 20 * index + 4 * digit]$                                                                         | $Mem[Mem[univ + 8 * index] + 4 * digit]$                                                                      |
| Single memory access                                                                                        | Double access to memory                                                                                       |

## Quiz 2 Prep

- Callee saved registers:  $\%rbx, \%rbp, \%r12, \%r13$  (must save/restore if modified)
- Caller saved registers:  $\%rax, \%rcx, \%rdx$ . (caller assumes will be altered by callee)
- Stack grows from higher memory addresses to lower
- $\text{addq}$  → increases stack pointer, but moves it UP, shrinking overall stack
- $\text{subq}$  → decreases stack pointer, moves to lower address, means stack expansion
- $\text{leaq}$  is used to compute memory addresses

## Q2 Stuff

Key addressing modes

$\$ \text{value}$  → value

$\% \text{register}$  → value @ reg

$(\% \text{register})$  → value @ memory address → value

$4(\% \text{rax})$  → adds 4 to value in rax, then use memory address to find result

$(\% \text{base}, \% \text{index}, \text{scale})$  →  $\text{base} + \text{index} * \text{scale}$  gives address then find value

displacement( $\% \text{base}, \% \text{index}, \text{scale}$ ) → address is  $\text{base} + \text{index} * \text{scale} + \text{displacement}$ , use address to get value

1)

$\text{movq } 1(\% \text{rcx}, \% \text{rdx}, 8), \% \text{rdi}$

$\text{movq} = \text{move 8 bytes from memory to register}$

$1(\% \text{rcx}, \% \text{rdx}, 8)$

$$= \% \text{rcx} + 1 + (\% \text{rdx} * 8)$$

$$= 0xFF + 1 + (0x01 * 8)$$

$$= 0xFF + 1 + 0x08$$

$$= 0x108 \rightarrow 0x7011, \text{dest} \rightarrow \% \text{rdi}$$

2)  $\text{addq } \% \text{rdx}, 6(\% \text{rax}, \% \text{rsi})$

- adds 0x01 to →

$6(\% \text{rax}, \% \text{rsi})$

$$= 6(0x110, 0x0A)$$

$$= 6 + 0x110 + 0x0A$$

$$= 0x120 \rightarrow \text{add } \% \text{rdx}$$

$$\text{Dest: } 0x120, \quad 0x220A + 0x01 = 0x220B$$

3)  $\text{subq } -16(\% \text{rax}), \% \text{rcx}$  4)  $\text{leaq } 0x30(\% \text{rax}, \% \text{rdx}, 8), \% \text{rsi}$

dest:  $\% \text{rcx}$

$$= \% \text{rax} - 16$$

$$= 0x10 - 16$$

$$= 0x100 \rightarrow 0x0055$$

$$\% \text{rcx} = 0x0055$$

$$0xFF - 0x0055 = 0xAA$$

$$0x110 + 0x01 * 8$$

$$0x30(0x118)$$

$$\text{dest} = \% \text{rsi}, \quad 0x30 + 0x118$$

$$= 0x148$$

5)  $\text{andq } \$0x0F, (\% \text{rax})$

dest: 0x110

perform Bitwise AND

$$0x0144 \text{ and } 0x000F = 0x0004$$

memory at 0x110 is 0x0004

## Q2 Practise

- if register is being modified, update register value
- if memory is being modified, update memory location

mova → Loads from memory → register  
 adda → adds to memory or register  
 suba → subtracts from memory or register  
 lea → compute address  
 anda → Bitwise AND

1) `movq %rdi, 1(%rcx, %rdx, 8)`  
 $1(\%rcx, \%rdx, 8)$       Value moved %rdi = 0x08  
 $= 1 + (0xFF + 0x08)$   
 $= 0x108$   
 Dest: 0x108, value: 0x08

2) `ora $0x0F, 0x10(%rax)`  
 or                      0x10 + 0x110  
 0000 0000 0000 1111 = 0x120  
 0010 0010 0000 10100 x220A  
 0010 0010 0000 1111

value = 0x220F  
 Dest: 0x120      ← add

2) `addq 6(%rax, %rsi), %rsi`  
 Dest: %rsi  
 value:  $6(\%rax, \%rsi)$   
 $= 6 + 0x110 + 0x02$   
 $= 0x118$   
 $= 0x00AB$  + add %rsi value  
                     ↳ 0x00AB + 0x02  
 Subtract ↓      = 0x00AD

3) `subq %rdx, -16(%rax)`  
 dest:  $-16(\%rax)$   
 $-16 + 0x110$   
 $= 0x100$  contains 0x0055  
 $0x0055 - 0x01 = 0x0054$   
 Dest: 0x100  
 Value: 0x0054

If operation is in form of:

`movq %register, memory address`      Dest = memory address  
`addq memory, %register`      Dest = Register

`ora $0x0F, %rax` - Bitwise of with %rax  
`ora $0x0F, (%rax)` - Bitwise or with Value in memory of %rax

4) `leaq 0x50(%rcx, %rdx, 8), %rsi`  
 dest: %rsi  
 value:  $0x50 + 0xFF + 0x01 \times 8$   
           : 0x197  
 Computes memory address

| Memory Address | Value  | Register | Value |
|----------------|--------|----------|-------|
| 0x200          | 0x0050 | %rax     | 0x200 |
| 0x204          | 0x0040 | %rbx     | 0x5   |

1) `addq %rbx, 4(%rax)`  
 Dest is a memory address  
 Content is going to be %rbx + 4(%rax)  
 Value =  $4 + 0x200$       Dest: 0x5  
           =  $0x204 + \%rbx$   
           ↳  $0x0040 + 0x5$   
           = 0x0045

Q2 - PRACTISE

| Memory Address | Content | Register | Content |
|----------------|---------|----------|---------|
| 0x100          | 0x0030  | %rax     | 0x100   |
| 6x104          | 0x0040  | %rbx     | 0x3     |
| 0x108          | 0x0050  | %rcx     | 0x4     |
| 0x10C          | 0x0060  | %rdx     | 0x1     |

- addq %rbx, (%rax)

Dest: 0x100

Value:  $0x3 + 0x0030 = 0x0033$
- movq %rcx, 4(%rax)

Dest: 0x104

Value: 0x4
- leaq 12(%rax, %rdx, 4), %rdi

$12 + 0xFF * 0x04 = C + 0xFF * 0x04 = 0x1F$

Dest: %rdi

Value: 0x1F
- andq \$0xf0, (%rax)

Dest: 0x110

Perform and with

|      |      |
|------|------|
| 0111 | 0000 |
| 1001 | 0000 |
| 0001 | 0000 |

Value: 0x10
- subq %rdx, 8(%rax)

Dest: 0x108

Val: 0x1
- movq %rcx, (%rax)

Dest = 0x110

Value = 0xFF
- addq \$0x03, (%rax)

Value:  $0x03 + 0x110 = 0x113$

Dest: 0x110
- subq %rsi, -8(%rax)

Dest:  $0x108 - 3 + 0x110 = 0x108$

Value:  $0x7011 - 0x0A$

1) movq 1(%rcx, %rdx, 8), %rdi

Dest: %rdi

Value:  $0xFF + (0x01 * 8) + 1$

$= (0x108)$

$= 0x7011$

2) addq %rdx, 6(%rax, %rsi)

Dest: 0x120

$0x110 + 0x0A + 6 = 0x120$

Value:  $0x01 + 0x220A = 0x220B$

3) subq -16(%rax), %rcx

Dest: %rcx

Value:  $0x110 - 0x10 = 0x100 = 0x0055 = 0xFF - 0x0055 = 0xAA$

4) leaq 0x30(%rax, %rdx, 8), %rsi

Dest: %rsi

Value:  $0x30 + 0x10 * 0x8 = 0x148$

5) subq 0x10(%rax), %rsi

Dest: %rsi

Value:  $0x10 - 110 = 0x100 = 0x0055$

7) movq 4(%rax, %rdx, 4), %rcx

dest: %rcx

$= 0x110 + 0x04 * 4 = 0x118 = 0x00AB$

6) addq (%rsi), %rax

$= %rax$

$0x02 + 0x110 = 0x112$

Q3

m x n matrix: 1280 bytes

Salsa \$6, %rdi, shift left %rdi (i) by 6 bits

- $\rightarrow %rdi = i * 2^6 = i * 64$
- $\rightarrow$  Each row takes up 64 bytes
- $\rightarrow$  Each row takes 64 bytes, each element has 4 bytes,  $\frac{64}{4} = 16$  is each row has 16 elements. # of cols is 16

Elements in Matrices

- char A[M][N] - 1 byte
- short A[M][N] - 2 bytes
- int A[M][N] - 4 bytes
- double A[M][N] - 8 bytes
- long A[M][N] - 8 bytes

movl A(%rdi, %rsi, 4), %eax, loading 4 byte (1=long) element into %eax

2560 bytes

$$A + (%rdi) + (%rsi * 4)$$

$$A + 64i + 4j$$

$$\%eax = A + 64i + 4j = A + 4(16i + j)$$

confirms each row has 16 cols

# %rdi =  $i * 2^5 = 32i$

#  $B + 32i + 8j = B + 8(4i + j)$

movq B(%rdi, %rsi, 8), %rax

$\rightarrow$  # of bytes per elem

# of rows =  $\frac{\text{memory}}{\text{cols} \times \text{elem size}}$

$= \frac{2560}{4 \times 8} = 10$

$N = \frac{\text{row size}}{\text{elem size}}$

rows = 80  
cols = 4  
elem = 8

Q3 v2

1280 bytes

salav \$6, %rdi

# %rax = Shift  $2^6 \cdot i = 64i$

movav A(%rdi, %rsi, 8), %eax

# %eax =  $A + 64i + 8j$   
 $= A + 8(8i + j)$

ret

Longs are 8 bytes:  $\frac{64}{8} = 8$

# of cols = 8

# of rows =  $\frac{1280}{8 \times 8} = 20$

# of size elem = 8

num of rows =  $\frac{\text{Total memory}}{\text{row size}} = \frac{\text{Total memory}}{\# \text{ num cols} \times \text{elem size}}$

$\%rdi = i \times 8 = 8i$

elem size = 4

$\%eax = A + 32i + 4j$   
 $= A + 4(8i + j)$

$\frac{1280}{16 \cdot 4} = 20$

### Q4 - Interpreting C vs Assembly

C code

```
long loop_1(long x, long y)
{
```

```
  long z = 1  # z is default set to 1  z = 1
```

```
  while (x <= y) { # condition
```

```
    z = z * (x + y) # z is calculated, using x and y, assigned new val
```

```
    x = x + 1 # x is incremented
```

```
  }
```

```
  return z;
```

```
  }
```

```
  For x = 1, y = 3
```

z = 1 \* 4

z = 4, x = 2

z = 4 \* (5)

z = 20, x = 3

z = 20 \* (6)

z = 120

Note: z does not change and only is returned once

Assembly code

loop\_1:

# Function name

movl \$1, %eax

rax = z = 1

# moves 1 to %eax register which holds z initially

jmp .L2

jump to check condition

# checks condition, due to while loop behaviour

.L3:

Starting point for loop

leaq(%rdi, %rsi), %rdx

rdx = z + y

# remember leaq just computes address

imulq %rdx, %rax

rax = z \* (z + y)

# refreshes on imulq

multiplies signed numbers

addq \$1, %rdi

rdi = x + 1

# increment x

# rax = rax \* rdx

.L2:

cmpq %rsi, %rdi

compare x and y (x - y)

# compares values in registers rsi and rdi and sees if x - y = 0

jle .L3

Loop when x <= y

# if x - y > 0, jump to .L3, Jle

ret

returns function

means jump if less than or equal to

# 4,5 - More advanced Translation

## C Code

```

long branch(long x, long y, long z){
    long val = x + y + z
    if ( x < 5 )
        if ( y < z )
            val = x * y ;
        else
            val = y * z ;
    }elseif ( x > 10 )
        val = x * z
    }
    return val;
}

```

```

cmpa $8, %rdi
jge .L2      means If x < 8
              continue

```

order matters in your operand

## Assembly Code

(x in %rdi, y in %rsi, z in %rdx)

branch:

```

leaq (%rdi, %rsi), %rax      longval = %rax = x + y      ✓
addq %rdx, %rax             longval = %rax = x + y + z      ✓
cmpa $5, %rdi               } If x ≥ 5 then jump to L2 or x < 5 ✓
                             }                               } continue
* jge .L2                   }
cmpa %rdx, %rsi             } If y ≥ z, then jump to L3 or y < z ✓
* jge .L3                   }                               } continue
movar %rdi, %rax            rax = x
imulq %rsi, %rax           rax = x * y
ret

.L3:
movar %rsi, %rax           longval = y
imulq %rdx, %rax          longval = longval * z
ret

.L2:
cmpa $10, %rdi             } If x ≤ 10, jump to L4 x > 10, continue
jle .L4                   }
movq %rdi, %rax            } else: longval = x
imulq %rdx, %rax           } longval = longval * z

.L4:
ret

```

(a in %rdi, b in %rsi, c in %rdx)

```

branches
    leaq 0(%rdi,%rsi), %rax
    addq %rdx, %rax
    cmpl $10, %rdi
    jge .L2
    cmpl %rdx, %rsi
    jge .L3
    movq %rdi, %rax
    imulq %rsi, %rax
    ret
.L2:
    movq %rsi, %rax
    imulq %rdx, %rax
    ret
.L3:
    cmpl $15, %rdi
    jle .L4
    movq %rdi, %rax
    imulq %rdx, %rax
.L4:
    ret

```

%rax = a + b ✓  
%rax = %rax + c ✓

} If a < 10, then continue, else jump to L2 ✓

} If b < c, then continue, else jump to L3 ✓

%rax = a ✓  
%rax = %rax \* b ✓  
return

%rax = b ✓  
%rax = %rax \* c ✓  
return

10? ✓  
If a > 15, continue, else jump to L4

%rax = a ✓  
%rax = %rax \* c ✓

return

```

long branch(long a, long b,
long c){
    long val = a + b + c; ✓
    if ( a < 10 ) { ✓
        if ( b < c ) { ✓
            val = a * b; ✓
        }
        else {
            val = b * c; ✓
        }
    }
    elseif ( a > 15 ) { ✓
        val = a * c; ✓
    }
    return val;
}

```



Assembly code  
(x in %rdi, y in %rsi)

```

Loop_1:
movl $1, %eax
jmp .L2
L3:
leaq(%rdi, %rsi)
imulq %rdx, %rsi
addq $1, %rdi
L2:
cmpl %rsi, %rdi
jle .L3
ret
    
```

(x in %rdi, y in %rsi)

%eax = 1

Jump to L2 (loop condition)

%rdx = x + y ✓  
 %rax = %rax \* (x + y) ✓  
 x = x + 1 ✓

Compare x and y  
 loop when x ≤ y

return

long Loop\_1(long x, long y)

2, 4

```

{
long z = _____; 1
while ( _____ ) { (x ≤ y)
z = _____; z * (x + y)
x = _____; x = x + 1
}
return z;
}
    
```

z = 1,

z = 1 \* (6)

z = 6

x = 3

z = 6 \* (7)

z = 42

x = 4

z = 42 \* (8)

z = 336

### Q3/04 Loops

#### C code

```

long loop_2(long x, long y) {
    long z = 1
    for (x = 1 ; x <= y ; x++) {
        z = z * (x+y)
    }
    return z
}

```

#### Assembly code (x in %rdi, y in %rsi)

```

%eax = 1
Compare x > y
loop when x > y, L2

.L3: %rdx = x+y
     %rax = %rax * (x+y)
     x = x+1
     Condition: x <= y
           go back up to .L3

.L2: ret This is return code or the stop
     Condition for the for loop.

```

Inside for loop

#### C Code

```

long loop_3(long x, long y) {
    long z = 1
    do {
        z = z * (x+y)
        x = x+1
    } while ( x <= y );
    return z;
}

```

#### Assembly code

```

%eax = 1
.L3: %rdx = x+y
     %rax = z * (x+y)
     x = x+1
Compare x and y
Jump to .L3
if x <= y

```

# Add/subtract operations

Case 1: dest is register

Value is added/subtracted to registers content, stored back  
 $FF - 55 = AA$

Case 2: Dest is memory

Find memory, read old value, add/subtract content to it and store back result

add/sub value, register

This +/- that

1) Dest: %rdi ✓  
 Value:  $1 + 0xFF + 0x01 * 8$   
 $: 0x108 \rightarrow 0x7011$  ✓

2) Val: 0x01 ✗  
 Dest:  $6 + 0x110 + 0x0A$   
 $= 0x120$  don't add content to address → Value  
 $=$

@  $0x120 = 0x220A$   
 Then add 0x01 to that get  $0x220B$

3) subw -16(%rax), %rcx  
 Dest: %rcx ✓  
 Val:  $-16 + 0x110$   
 $= 0x100 - 0xFF$   
 $= 0x01$  ✗ 0x0055,  $FF - 55 = AA$

4) leaq 0x30(%rax, %rdx, 8), %rsi  
 Dest: %rsi ✓  
 Val:  $0x30 + 0x110 + 0x08$   
 $: 0x148$  vs.  $0x17C$

5) andq \$0x0F, (%rax)  
 0x0144

0000 0000 1111  
 0001 0100 0100  
 0000 0000 0100  
 $= 0x04 \rightarrow 0x004$  ok ✓  
 Dest: 0x110 ✓

1) Dest:  
 Val:  $0x08$  ✓  
 $1 + 0xFF + (0x01 * 8)$   
 $= 0x108$   
 $= 0x7011$  ✓  
 Dest: 0x108  
 Val: 0x08

2) Dest: %rsi  
 Val:  $6 + 0x110 + 0x02$   
 $= 0x118$  ✓  
 $= 0x00AB + 02$   
 $= 0x00AD$  ✓

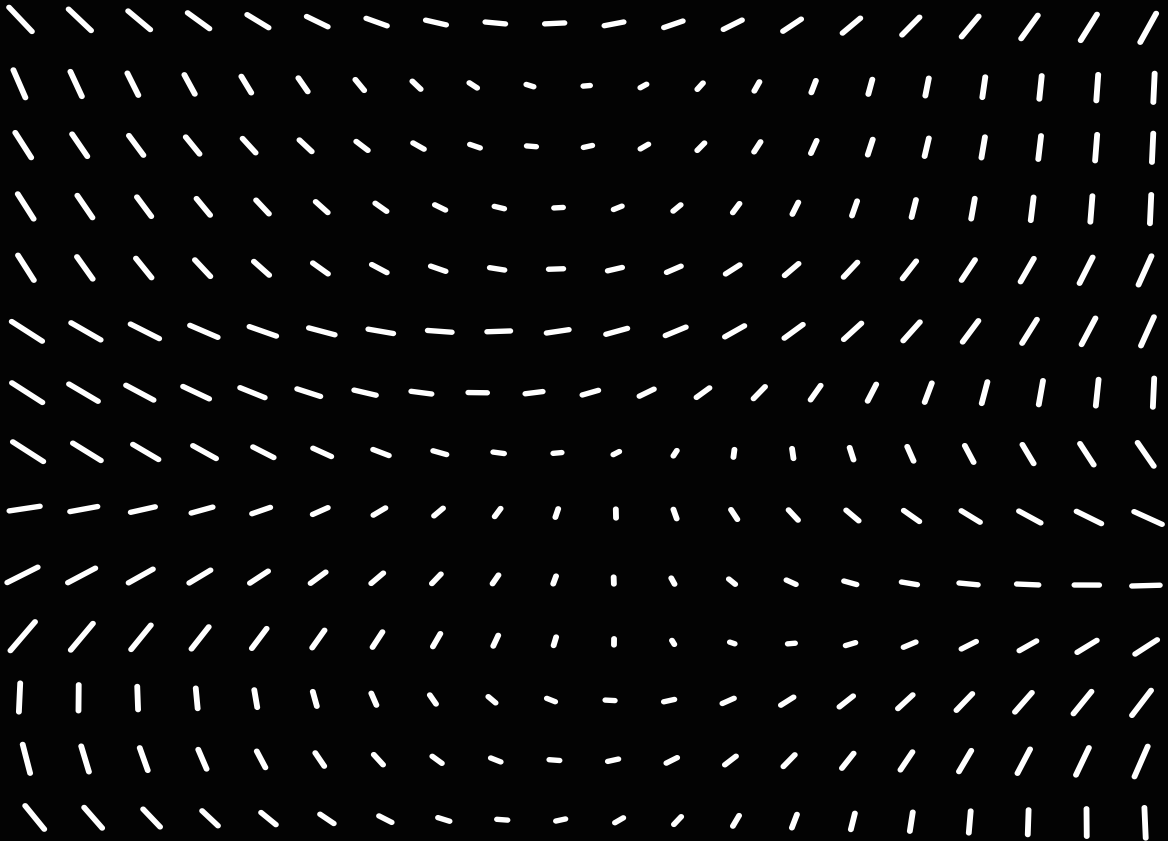
3) Dest:  $-16 + 0x110$   
 Dest:  $= 0x100$  ✓  
 $= 0x0055$   
 $0x0055 - 0x01$   
 $= 0x0054$  ✓

4) Dest: %rsi ✓  
 Val:  $0x50 + 0xFF + 0x08$   
 $= 0x157$  ✓

5) Dest:  $0x0144 + 0x10$  0x120 → 0x220A  
 $= 0x154$  ✗

Val:  
 0000 0000 0000 1111  
 0010 0010 0000 1010  
 0010 0010 0000 1111  
 $= 0x220F$

# Unit 3



# Lecture 15 - Machine Structure

## Structure

- a Struct in C is a block of memory that contains multiple fields
- Fields are ordered as declared
- Compiler determines size and positions of fields

## Memory Alignment

- Aligned memory speeds up access and avoids CPU inefficiencies

Ex

```
struct {  
    char c;      1 byte - compiler adds padding to satisfy alignment  
    int i[2];    8 bytes  
    double v;    8 bytes  
} *p;
```

- For primitive datatype requiring X bytes the address must be multiple of X (2, 4, 8)

## Arrays of Structures

- When creating an array of structures, each structure must be properly aligned
- Each structure in the array is 98 bytes as compiler adds padding
- Compiler calculates offsets based on alignment, alignment affects memory layout
- Save space by reordering struct initializers

## Unions

- Union allows different variables to share same memory (but only one field at a time)

## Byte Ordering

- Big Endian (MSB First)    12 34 56 78    (space)
- Little Endian (LSB First)    78 56 34 12
- modern CPU's use little-endian

## Floating Point Operations

- Floating point registers: XMM0 to XMM15 (can store 4 single-precision / 2 sp floats)
- FP memory referencing, arguments passed in regular registers
- FP values passed in XMM registers

# Lecture 16 - Digital logic: logic gates

## General Intro

- Computers are built on multiple abstraction layers
- Highest level (software side)
- Lowest level (hardware side)
- Each layer depends on one below it.

### • ISA (Instruction Set Architecture)

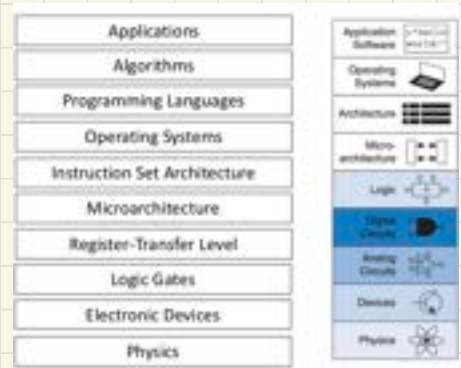
↓  
The set of instructions a CPU understands

- Micro Architecture → design of actual CPU that executes those instructions

- Hardware/Software must evolve together
- Digital logic gates dictate limits of hardware design.
- as transistors shrink, we hit physical limits.

### • Logic gates (And, or, Not) are foundation of binary computation

- CPU's built using millions logic gates are complex circuits
- efficiency of gates directly impact processor speed and power consumption



## Digital Logic Circuits (DLC)

- DLC are built using transistors in chips
- Transistors act as a simple switch that create logic gates (And, or, not)
- Boolean expression describe circuit's function
- Complex circuits are built using networks of simple logic gates
- all digital devices are built using billions of transistors to form logic gates

## Primitive Logic Functions

- logic gates take binary inputs (0 or 1) and produce binary output
- gates are represented using symbols

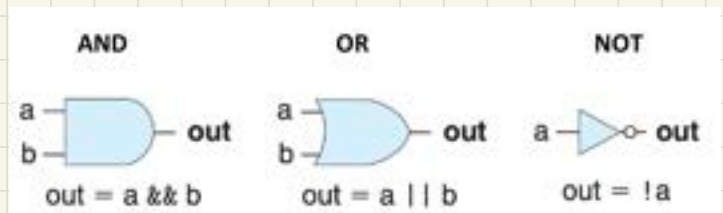
Truth tables → all possible input/output combos

boolean expressions → math version of logic

$\wedge$  - and

$\vee$  - or

$\neg$  - negation (not)



## NAND and NOR Gates

- NAND - Not and, negates output of A and B
- NOR - Not OR, negates output of A or B
- Drawn with small circle (inverter) at output
- Note: Every other gate can be made using just NANDS

## Multiple Input Gates

- Some gates take more than 2 inputs: (a, b, c)
- NOR3 gate outputs 1, when a, b, c is 0.
- more inputs  $\rightarrow$  more control over logic operations

## Bit Adder

- bit adder is a digital circuit used to add binary numbers
- Full adder takes 3 inputs, two outputs
- $S_1 = a_1 \oplus b_1 \oplus c_1$
- $C_{1+1} = (a_1 \wedge b_1) \vee (b_1 \wedge c_1) \vee (a_1 \wedge c_1)$
- if both bits and carry are 1, results in carry out 1.

$a_1$  = first bit

$b_1$  = second bit

$c_1$  = carry in (previous carry)

$S_1$  = sum bit

$C_{1+1}$  = carry out bit

# Lecture 17 - Combinational Circuits

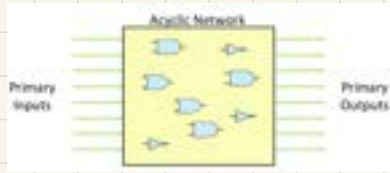
## Combinational Circuits Overview

- Combinational circuits are LC, where output determined by current input values
- functional Specification: describes relationship between inputs and outputs (using boolean expressions)
- timing Specification: defines delays between input changes and output updates
- no memory / prev state
- can use diff combinations to get same result
- single / line means number indicates a bus

## Rules of Combinational Circuit Composition

- Circuit is combinational if:
  - 1) Each circuit is itself combinational
  - 2) Each node in circuit is either input or connected to only one output
  - 3) No feedback loops

Note: Loops create sequential circuits, which need memory elements, which is sequential logic



## How to build Combinational circuit

- 1) create truth table
- 2) Boolean expressions based on T.T
- 3) Build circuit using logic gates to implement boolean functions

### Terminology

- Literal: a variable or its complement (inverse)
- Product (or implicant): the AND of one or more literals
- Minterm: a product involving all inputs to the function
- Sum: the OR of one or more literals
- Maxterm: a sum involving all inputs to the function

EX  $Y = AB + A'B$   
Means  $A=1, B=1$ , or  $A=0, B=1$

### SOP Form

- way to express boolean functions using Anded minterms that are OR-ed together
- write terms with output as  $Y=1$  as minterms

EX

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

minterms: where  $Y=1$  are

$$\begin{aligned} &A'B \\ &AB' \\ &AB \end{aligned} \quad \therefore Y = A'B + AB' + AB$$
$$Y = \sum (m_1, m_2, m_3)$$

## From Logic to Circuits

• any SOP equation can be drawn as a circuit

- Steps:

- 1) Draw input variables
- 2) Add NOT gates for complemented variables
- 3) And gates for min terms
- 4) Or gate to combine all min terms.

• Note: Not always easiest circuit

## Boolean Algebra Axioms

• Axioms define how binary variables behave

| Axiom | Def                     | Name         |
|-------|-------------------------|--------------|
| A1    | $B=0 \text{ or } B=1$   | Binary field |
| A2    | $\bar{\bar{A}}=A$       | NOT          |
| A3    | $0 \cdot 0=0$           | AND/OR       |
| A4    | $1 \cdot 1=1$           | AND/OR       |
| A5    | $0 \cdot 1=1 \cdot 0=0$ | AND/OR       |

↳ Allows us to simplify logical expressions

| Theorem | Def               | Name         |
|---------|-------------------|--------------|
| T1      | $A+1=A$           | Identity     |
| T2      | $A+0=A$           | Null Element |
| T3      | $A+A=A$           | Idempotence  |
| T4      | $\bar{\bar{A}}=A$ | Inversion    |
| T5      | $A+\bar{A}=1$     | Complement   |

## De Morgan Theorem

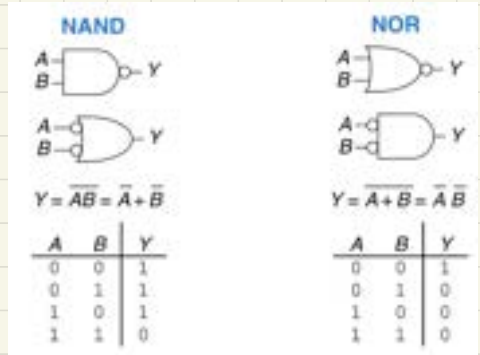
• Helps convert between and and or operations using inverters (not gates)

1.) NAND  $\rightarrow$  OR with inversion

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

2.) NOR  $\rightarrow$  And with inversion

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$



## Simplifying Boolean Expressions

- goal is to reduce number of terms and logic gates
- SOP expression is minimized, with fewest terms
- require fewer logic gates

EX:  $\bar{A}\bar{B}C + A\bar{B}C + ABC + \bar{A}BC$

$$= \bar{A}\bar{B}C + A\bar{B}C + ABC$$

$$= \bar{B}C(1) + ABC(1)$$

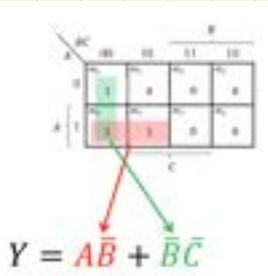
$$= \bar{B}C + AB$$

# Karnaugh Maps

- Visual for boolean expression, best up to 4 variables
- arranges minterm in grid
- Default Setup:

|                |                |                |                |
|----------------|----------------|----------------|----------------|
| m <sub>0</sub> | m <sub>1</sub> | m <sub>3</sub> | m <sub>2</sub> |
| m <sub>4</sub> | m <sub>5</sub> | m <sub>7</sub> | m <sub>6</sub> |

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |



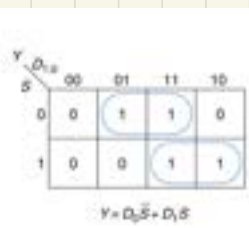
- 1) Find minterms, put them on grid
- 2) group adjacent 1's together (group size, 1, 2, 4) (bigger group minimize more terms)
- 3) Each group is a product term

## Don't Care Conditions

- Some inputs do not affect the output
- marked as x
- Simplifies logic, reduces number of logic gates
- Priority circuit is a combinational circuit, determines which input has highest priority
- only highest priority active input determines output

## Multiplexors Introduction

- A multiplexor is a combinational circuit that selects one of multiple input signals and forwards it to output
- Uses a select signal (S) to decide which input to pass
- efficiently control data routing

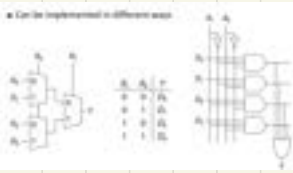


| S | D <sub>1</sub> | D <sub>0</sub> | Y |
|---|----------------|----------------|---|
| 0 | 0              | 0              | 0 |
| 0 | 0              | 1              | 1 |
| 0 | 1              | 0              | 0 |
| 0 | 1              | 1              | 1 |
| 1 | 0              | 0              | 0 |
| 1 | 0              | 1              | 0 |
| 1 | 1              | 0              | 1 |
| 1 | 1              | 1              | 1 |

2 Data inputs D<sub>0</sub>, D<sub>1</sub>  
 1 Select line S  
 output Y  
 $Y = D_0 \bar{S} + D_1 S$

Note: Not gate to invert S

## 4:1 Multiplexor



- 4 data inputs
- 2 select lines ( $S_1, S_0$ )
- 1 output ( $Y$ )

$$Y = D_0 \bar{S}_1 \bar{S}_0 + D_1 \bar{S}_1 S_0 + D_2 S_1 \bar{S}_0 + D_3 S_1 S_0$$

## Decoder

- A decoder has  $N$  inputs and  $2^N$  outputs
- it activates only one output based on binary input
  - used in address decoding in memory
  - instruction decoding in CPU's
- decoder generates minterms

## Enabled Decoder

- Enable ( $E$ ) input controls decoder activation
- When  $E=1$ , decoder works normally
- When  $E=0$ , all outputs are 0.

## Timing

- Circuits don't change instantly, output has delay for input change
- measure timing at 50% transition point
  - Rising edge (Low  $\rightarrow$  High transition)
  - Falling edge (High  $\rightarrow$  Low transition)
- Propagation delay ( $T_{pd}$ ) = Maximum time an output takes to stabilize
- Contamination delay ( $T_{cd}$ ) = Minimum time before any output starts to change
  - to calculate  $T_{pd}$  and  $T_{cd}$  of gates looking into internals
- Delays are caused by charging/discharging capacitance, different rise/falling delays
- Critical path:
  - the longest (slowest) path is the critical path
  - The critical path delay determines how fast circuit operates
- Shortest path:
  - related to contamination delay, how fast any change in input starts affecting outputs
- Glitches:
  - Signal takes multiple paths to output, temporary incorrect outputs
    - to solve glitches, adding delay buffers or rearranging logic gates
  - Faster processors minimize critical path delays

# Lecture 18 - Digital Logic ALU

## Building Combinational Circuits

- CC produce outputs only based on current inputs

Process:

- 1) Truth table
- 2) Boolean Expression
- 3) Logic Circuit
- 4) K-map Simplify
- 5) Gate optimization & best timing (fewer gates doesn't directly mean fastest due to prop delays)

## ALU (Arithmetic Logic Unit)

- type of combinational circuit that performs arithmetic (add/subtract, and, or)
- Heart of CPU

## Full Adder vs 1 bit Adder

- 1 bit adder computes sum and carry out
- Full adder, adds two inputs A and B, carry in, generates sum and carry out.
- Circuit diagrams use xor gates for sum.

## Carry Propagate Adder

- Used for multi bit numbers
- Carry out of one stage propagates to next bit
- ripple carry adder (Simpler but slow)
- carry-lookahead adder (faster) → pre computes bits (generates  $G_i = A_i B_i$ ,  $P_i = A_i + B_i$  (passes))
- prefix adder (Optimized)

## Comparator

- determines whether two binary numbers are equal or which is larger
- Equality:  $A = B$
- Magnitude:  $A < B$

## Shifter

- Shifter moves bits left/right
- 1) Logical Shift: fills shifted positions with zeros
- 2) Arithmetic Shift: preserves sign bit (MSB)
- 3) Rotator

- Decoder - Circuit changes Code to a set of signals
- Multiplexor - Several inputs to share one device

## Multiplier

- multiplication is done using repeated addition and shift operations

## Reading ALU

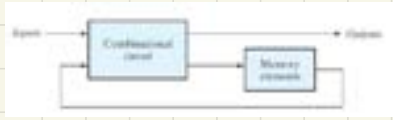
- inputs A, B
- operation selected using control signal  $F_{2,0}$
- inverter suggests subtraction

## Random Notes

- Latch: Level Sensitive (output changes when CLK is at 1)
- Flip-Flops: Edge Triggered (output only changes on a rising edge of CLK)

# Lecture 19 - Sequential Logic

- Output depends on both current inputs and past inputs
- Uses some kind of memory, sequence of inputs  $\rightarrow$  sequence of outputs



## Synchronous data processing

- CLC operations are a **synchronous**
- Synchronize data transfer/processing by **timed registers**  $\rightarrow$  ensures stabilized input to CLC
- Clock period should be greater than the CLC:  $\rightarrow$  ensure valid outputs

$$T_C > T_{pd}$$

$$\text{Period} = T_C = \frac{1}{\text{frequency}}$$



- updates only on the clock edge

## Bi-Stable Devices

- bi-stable device has two stable states, switches between them
- remains in one state until external force/energy applied to change it

## Cross Coupled Inverter Pair

- CCIP is an electrical circuit exhibits bi-stability. (part of memory circuits)
- Two outputs  $Q$  and  $\bar{Q}$  (output of one is input to other)

## SR Latch

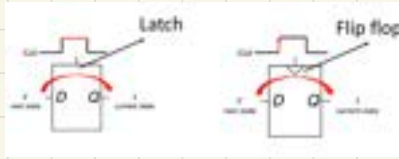
- An SR (Set-Reset) latch is bi-stable circuit used to store single bit.
- Two inputs S (set), R (reset). Controls  $Q$  and  $\bar{Q}$
- Built using cross coupled NOR gates
- Avoid state is  $(S=1, R=1)$  causes  $Q, \bar{Q}$  to be 0

## D-Latch

- D latch is modified SR Latch that eliminates invalid states
- Takes in Data (0/1) input and CLK input, Controls when value stored
- Latch is level sensitive, changes state when signal is high

## Level Trigger vs Edge Trigger

- Level triggered: Enable is High, latch continuously updates
- Edge triggered: latch only changes, when clock signal transitions
  - Positive edge:  $0 \rightarrow 1$
  - Negative edge:  $1 \rightarrow 0$



## D Flip-Flop

- Stores single bit of information
- has data and CLK input
- Unlike d-latch, D Flip flop updates, only on rising edge of clock
- D flip flop - edge triggered, 51-18
- Commonly used in state machines

## Register

- Collection of  $N$  flip-flops (share common CLK)
- Registers store multi-bit data update simultaneously
- Key components of sequential circuits

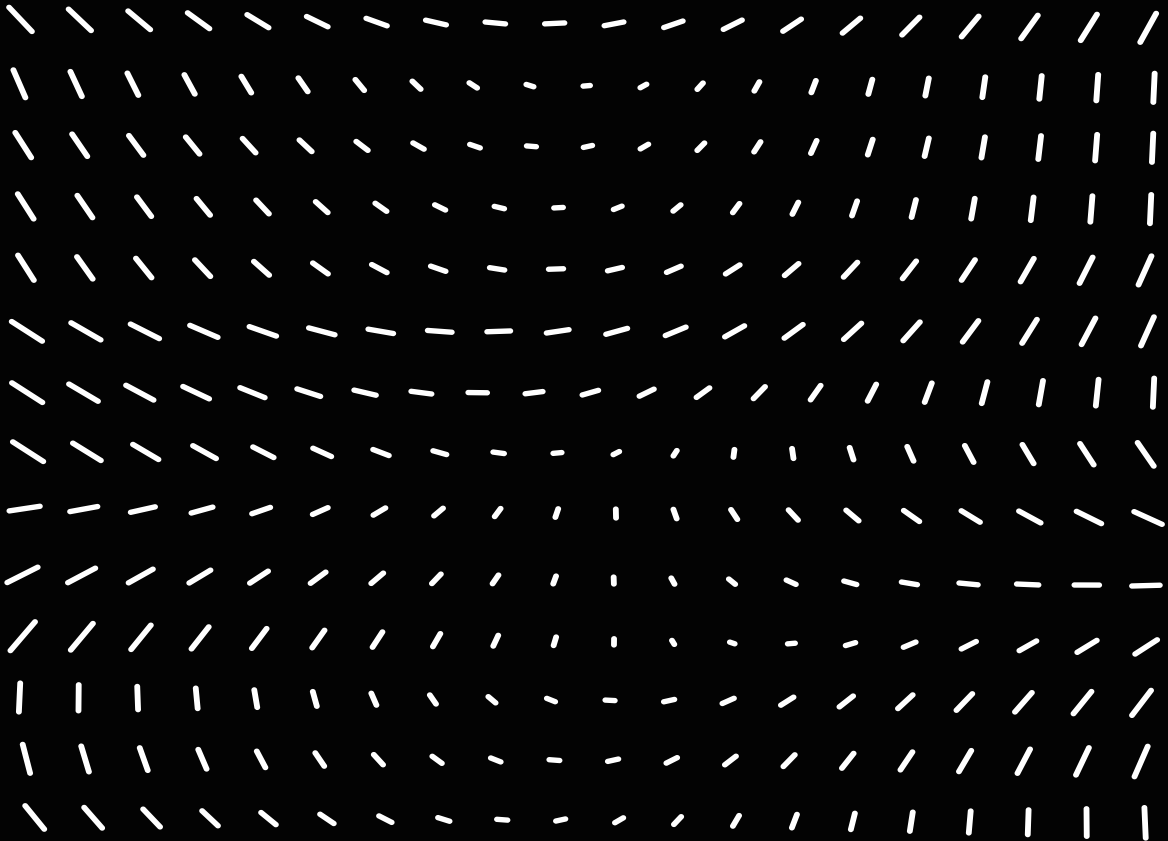
## Counter

- Binary counter is sequential circuit for counting
- CLK input, reset input,  $n$  bits outputs

## Shift Register

- Stores multiple bits and shifts through data
- on rising edge, new bit shifts in all prev bits move forward

# Practise Problems



Please Contact me for more info